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RESEARCH MEMORANDUM

EXAMINATION OF SOME MODELS OF FAILURE OF EQUIPMENT DURING OPERATION (U)

D. J. Davis
W. J. Howard

RM-87

26 October 1948

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SUMMARY

A theoretical study is made of some of the factors affecting ''chance'' distribution of failure of physical articles. (''Chance'' failure is defined as failure that has equal probability of occurrence to survivors during equal time intervals throughout the operating period). A very simplified model of mechanism and environment is set up as a point of departure. Several other more realistic, and more sophisticated, models are introduced and compared with the original simple case. It is shown that various extensions and alterations of chance failure curves describe reasonable combinations of mechanism and environment, provided the mechanism does not deteriorate in failure resistance (i.e., does not tend to wear out) during the operating period. Because of their short flight durations, guided missiles can be expected to fit this condition.

INTRODUCTION

A previous study of experimental data[1] have shown that many failures of equipment, particularly of the kinds associated with guided missiles, resemble a ''chance'' failure distribution as defined in the Summary, above. A detailed examination of this type of failure and the way in which it relates to ''wear out'' failure has been the next logical step in studying guided missile reliability.

The investigation of failure of equipment naturally involves a study of the characteristics of the mechanism in relation to operating environment. Interest must be focused on (a) the inherent resistance to failure of the article and the ways in which this resistance may vary and (b) the severity of the environment and the ways in which it may vary. Probably there are infinite conditions under which environment and resistance to failure can combine to give a "chance" distribution of failure, and an exhaustive mathematical study would be too involved for the present purposes. Instead, some particular relations between resistance and environment will be considered. In what follows, the selected sets of conditions are called ''models''.

A simplified model of ''chance'' failure would operate under the following conditions:

1. All the mechanisms are identical in failure resistance at the start.
2. The failure resistance of the mechanisms does not decrease with operating time.
3. Failure arises from the occurrence of a single event or one of several events, an instance of which is equally likely to happen at any time and thus have a fixed probability of occurrence per unit time.

The conditions of this model of failure appear oversimplified in the following respects:

Condition 1. Individual mechanisms are not likely to exhibit the same failure resistance but will have some sort of frequency distribution between the best and worst of the population. A fixed probability of failure per unit time, of course, results if an environment more severe than a particular value fails all the mechanisms and less than that severity value fails none of the mechanisms. With variability of failure resistance among mechanisms, the probability of failure per unit time varies among the different individuals.

Condition 2. Deterioration of failure resistance can be expected of at least some characteristics of a mechanism during operation. However, for the short periods of use in guided missiles the assumption seems valid. Deterioration prior to use can well be considered under condition 1 as producing a variable failure resistance among different members of the population.

Condition 3. Failure probably arises as the result of the failure-causing-environment exceeding the failure resistance of the particular mechanism. The environment could consist of a series of discrete events whose magnitude is distributed in some as yet unspecified fashion. A simple assumption is that the events occur at equal time spacings. More realistically their time spacings might be considered a random variable, having a particular mean value.

A continuously variable environment such as a radio-signal voltage can be evaluated as a series of discrete events, for the peak voltages may well impose the greatest strain on electrical components. The severity of each event is the peak voltage; the mean frequency of occurrence is determined by the average spacing of the peaks.

Mathematical Models

In view of the preceding remarks some realistic assumptions are made as to the distributions of environmental severity and failure resistance and models thereof examined.

A DEFINITION OF ENVIRONMENTAL SEVERITY AND FAILURE RESISTANCE. Before proceeding further with the examination of mathematical models of failure, the concept of environmental severity must be defined.

Failure of mechanisms results from a variety of environmental effects such as forces, heat, electrical phenomena, chemical reaction, etc., and all the ramifications of such physical occurrences. The whole spectrum of environmental effects are potential causes of failure for a particular mechanism which has a specific but different level of resistance to each effect.

It appears reasonable to assume that the various types of occurrences are independent events as are the mechanism's failure resistance to each of the types of environment. Failure of the mechanism is then the occurrence of any one of several independent events. The models examined assume a single type of environment and evaluate the resulting failure distribution with time. The failure distribution of an actual mechanism must account for all causes and is therefore a new distribution made up of those arising from each cause. Some typical cases of combined distributions have been discussed in Ref. 1.

In this discussion both failure resistance, R, and environmental severity, E, are evaluated in terms of the initial mean failure resistance of the population of mechanisms, making them dimensionless ratios:

$$R = \frac{\text{failure resistance of a particular mechanism}}{\text{initial mean failure resistance of the mechanisms}}$$

$$E = \frac{\text{severity of a particular environmental event}}{\text{initial mean failure resistance of the mechanisms}}$$

B. NUMBER OF ENVIRONMENTAL OCCURRENCES

In all the models the failure-causing environment will be considered as a series of discrete events of varying severity. The events are assumed to be random occurrences equally likely to happen at any time during the operating period; the mean time between events is designated $\bar{y}$.

The number of occurrences (n) exhibited during an operating period (t) follows a Poisson distribution in which the probability of precisely n occurrences, P(n), is given by the expression:

$$P(n) = \frac{1}{n!} \left(\frac{t}{\bar{y}}\right)^n e^{-t/\bar{y}} \quad \text{- - - - - - - - - - - (1)}$$

C. SEVERITY OF ENVIRONMENTAL OCCURRENCES

If the distribution function of the severity of discrete environmental occurrences is represented by f(E), then the probability of a single event not exceeding a particular value, E_1, is:

$$P_1 (E \leq E_1) = \int_0^{E_1} f(E)dE \quad \text{- - - - - - - - - - - (2)}$$

and the probability that none of a series of n events exceeds the value E_1 becomes

$$P_n (E \leq E_1) = \left[\int_0^{E_1} f(E)dE\right]^n \quad \text{- - - - - - - - - - - (3)}$$

During an operating period, t, the probability of no environment exceeding a given severity is the summation of the probability of getting each number of environmental events times the probability that none of that number exceeds the given value in severity:

$$P_t \quad (E \leq E_1) = \sum_{n=o}^{\infty} P(n) \quad P_n \quad (E \leq E_1)$$

$$= \quad e^{-t/\bar{y}} \sum_{n=o}^{\infty} \frac{1}{n!} \left[\frac{t}{\bar{y}} \int_o^{E_1} f(E) dE \right]^n$$

$$= \quad e^{-t/\bar{y}} \quad e^{t/\bar{y} \int_o^{E_1} f(E) dE}$$

$$= \quad e^{-t/\bar{y}} \left[1 - \int_o^{E_1} f(E) dE \right] \text{-------------------------------------(4)}$$

In passing it is recalled that the expression $1 - \int_o^{E_1} f(E)dE$ represents the probability of the environment having a value greater than E_1 during a single event which is considered a potential cause of failure. If the mechanism is to have a reasonable chance of surviving an operating period consisting of a large number of such events, it is obvious that the probability of failure per event must be very small. Expanding equation (4) in an infinite series, the third and subsequent terms may, under these special conditions, be disregarded as increasing powers of a very small quantity and:

$$P_t \quad (E \leq E_1) \doteq \left[\int_o^{E_1} f(E) dE \right]^{t/\bar{y}} = \left[P_1 \quad (E \leq E_1) \right]^{t/\bar{y}} = \left[P_1 \quad (E \leq E_1) \right]^{\bar{n}} \text{ - - (5)}$$

$t/\bar{y}$ is the expected or mean number of occurrences, $\bar{n}$, indicating that the conditions of the model are closely approximated by assuming a constant number of environmental events equal to the mean number occurring during the time interval. The approximation (Eq. 5) may be substituted for the exact expression (Eq. 4) if convenient under the conditions stated.

D. FAILURE RESISTANCE

As before stated the failure resistance of a mechanism in the model will be assumed to be the maximum environmental severity that the mechanism can withstand without failure. Failure resistance, which will be designated by R, will be in the same dimensionless units as environmental occurrence severity, E. The normal distribution is intuitively satisfying as a description of failure resistance of

a group of mechanisms and is used for all models. By previous definition the mean of the failure resistance distribution is unity; the standard deviation is designated by the symbol σ. From the normal distribution[2] the probability of failure resistance of a mechanism having a value between R_1 and $(R_1 + dR)$ is

$$P(R_1 < R < R_1 + dR) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left(\frac{R_1 - 1}{\sigma}\right)^2} dR \quad \text{- - - - - - - - - -} \quad (6)$$

E. PROBABILITY OF NO FAILURE FOR THE POPULATION

The probability of no failure for a particular mechanism having a failure resistance R_1 is obviously the probability that no environmental event exceeds R_1 in severity (Eq. 4). The probability of no failure of a randomly selected mechanism is then the integral over the range of R values of the probability of no environmental event being more severe than the value R_1 times the probability of getting a mechanism with the precise failure resistance R_1.

$$P = \int_{0\doteq-\infty}^{\infty} e^{-t/\bar{y}\left[1 - \int_0^R f(E)dE\right]} \left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left(\frac{R-1}{\sigma}\right)^2}\right] dR \quad \text{- -} \quad (7)$$

or

$$P \doteq \int_{0\doteq-\infty}^{\infty} \left[\int_0^R f(E)dE\right]^{t/\bar{y}} \left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left(\frac{R-1}{\sigma}\right)^2}\right] dR \quad \text{- - - - - -} \quad (8)$$

Since in the models selected the integral cannot be evaluated directly, a graphical method is employed. For corresponding values of R_1 ranging from 0 to ∞, a graph is plotted using $Q = e^{-t/\bar{y}\left[1 - \int_0^{R_1} f(E)dE\right]}$ as the ordinate value and

$S = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{0\doteq-\infty}^{R_1} e^{-1/2\left(\frac{R-1}{\sigma}\right)^2} dR$ as the abscissa value. Then an element of area under the curve is :

$$Q\,d\,S = e^{-t/y}\left[1 - \int_0^{R_1} f(E)\,dE\right] \cdot \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-1/2\left(\frac{R-1}{\sigma}\right)^2}\, dR$$

and the whole area under the curve is

$$\int Q\,d\,S = P \text{ - } (9)$$

By repeating this procedure for various values of the mean number of environmental occurrences, $\bar{n}$, corresponding to different lengths of the operating period, t, a curve of probability of success or reliability for a particular model can be related to the operating period.

F. MODELS EXAMINED

Model Ia. The environmental severity is assumed to have the negative exponential distribution $f(E) = \frac{1}{0.2}\, e^{-\frac{E}{0.2}}$ the mean value of E being 0.2. The least severe events occur most often and the frequency of occurrence decays exponentially as the severity increases, a trend that applies to many environmental features, for example, air gusts. The probability of the environmental severity of a single event being greater than unity (mean failure resistance of the mechanisms) is approximately 0.007.

The failure resistance of the mechanisms is assumed to have the normal distribution:

$$f(R) = \frac{1}{0.2\sqrt{2\pi}}\, e^{-1/2\left(\frac{R-1}{0.2}\right)^2}$$

for which the standard deviation, 0.2, produces a rather wide spread of values of R (95.5 percent lie between 0.6 and 1.4). The failure resistance was assumed not to deteriorate with use. Fig. 1 is a typical series of working curves, the area under each of which represents the reliability for a lifetime consisting of $\bar{n}$ mean number of environmental occurrences. In Fig. 2 reliability is plotted as a function of mean number of occurrences (operating time) using the values obtained from Fig. 1. The negative exponential character of the curve is obviously similar to the ''chance'' failure distributions found in actual data.

Model Ib. This model is identical to Model Ia except that the standard deviation of the mechanism failure resistance is $\sigma = 0.25$ and the frequency distribution of environmental severity,

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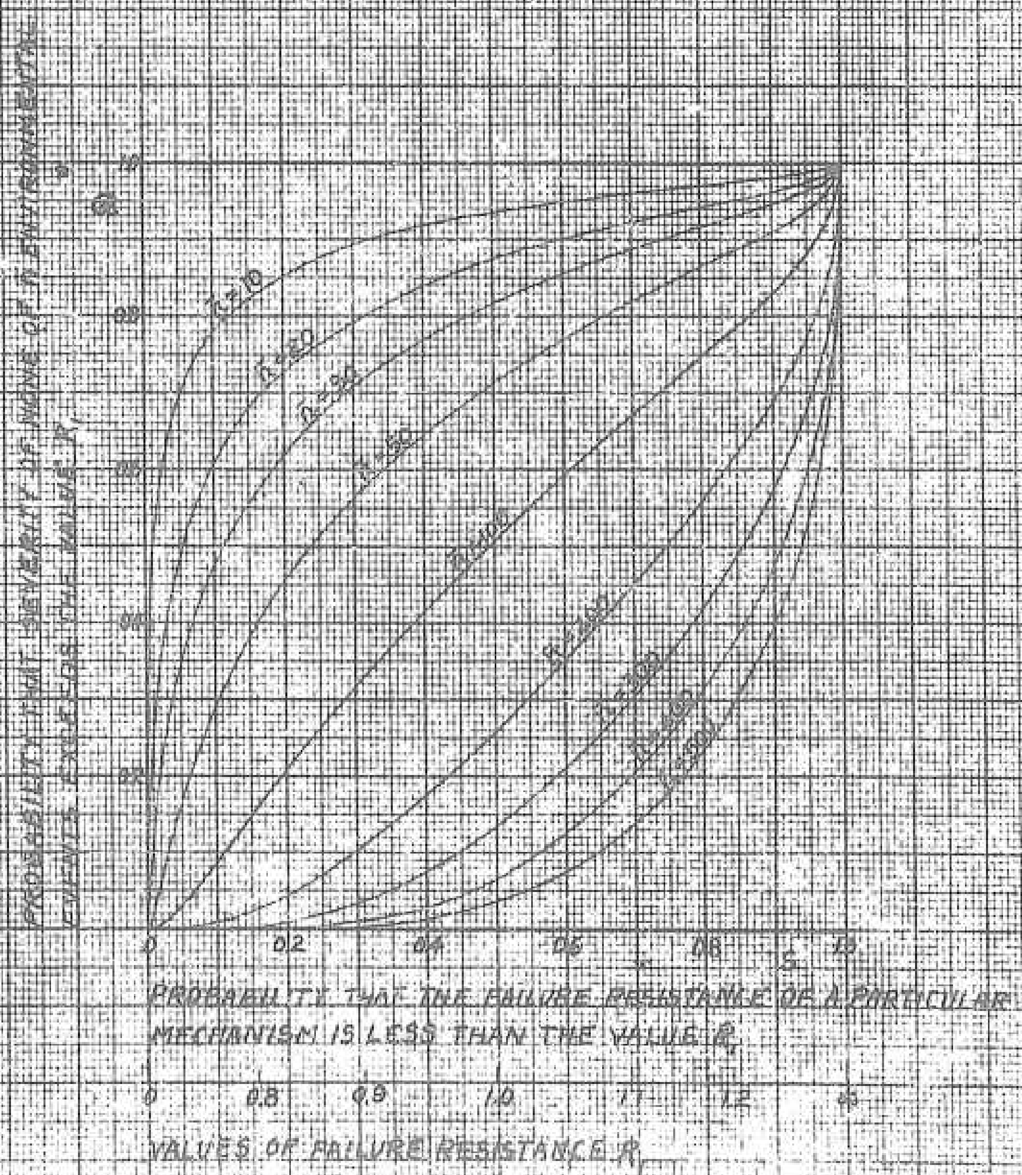
FIG. 2. GRAPHICAL METHOD OF EVALUATING PROBABILITY OF SUCCESS - MODEL II

Mean environmental severity $\bar{R} = 0.2$
Mean failure resistance of mechanism $\bar{R} = 1$
Standard deviation of failure resistance $\sigma = 0.1$
Mean number of environmental occurrences $= \bar{n} = \frac{1}{T}$

$$Q = e^{-\bar{n} e^{-R/0.2}}$$

$$S = \frac{1}{0.1\sqrt{2\pi}} \int_{-\infty}^{R} e^{-\frac{1}{2}\left(\frac{R-1}{0.1}\right)^2} dR$$



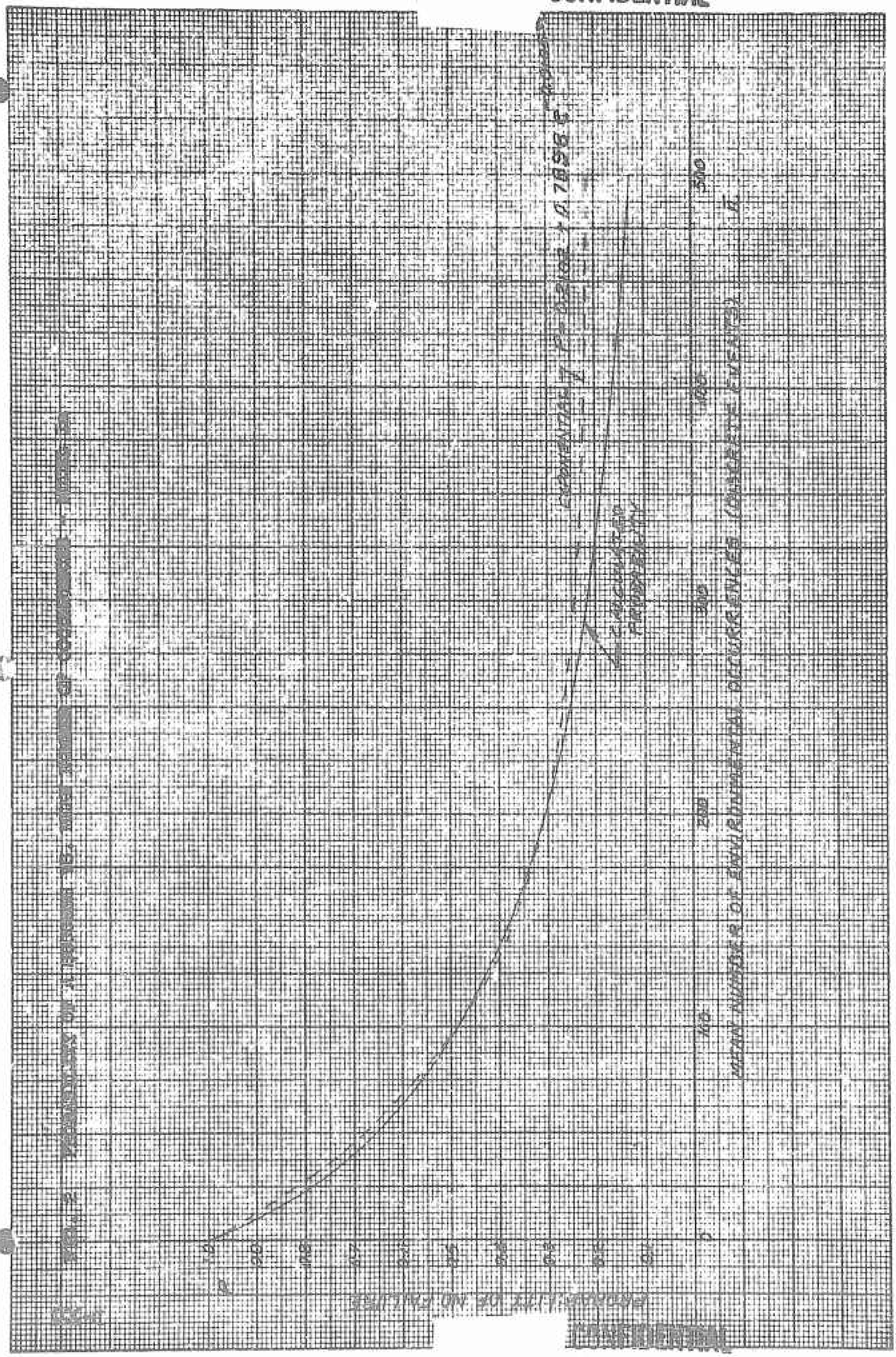


FIG. 2 [redacted]
PROBABILITY OF NO FAILURE
P
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
MEAN NUMBER OF ENVIRONMENTAL OCCURRENCES (DISCRETE EVENTS) $\bar{n}$
EXPONENTIAL $P = 0.2102 + 0.7898\,e^{-0.0\ldots\bar{n}}$
CALCULATED PROBABILITY

$$f(E) = \frac{1}{5} e^{-E/5}$$

The mean severity of environmental events ($\bar{E} \approx 5$) is then much greater than the mean failure resistance and the occurrence of an environmental event practically assures a failure. The probability of success is very nearly the probability that no environmental event occurs, and, as might be expected, the probability of success curve is only slightly higher than the exponential curve representing the probability of no events occurring when $\bar{n}$ events are expected (see Fig.3):

$$P = e^{-\bar{n}} \quad \text{where } \bar{n} = \frac{t}{\bar{y}} .$$

Model Ic. The region intermediate between the extremes of Models Ia and Ib is examined by choosing the mean environmental severity equal to the mean failure resistance. The standard deviation of failure resistance is 0.25 as in Ib. Figure 4 illustrates the exponential character of this failure probability intermediate between those of Figs. 2 and 3. The dotted curve is the exponential $P = e^{-\bar{n}/e}$.

Model II. The conditions of this model are identical to those of Model Ia except that environmental severity was assumed to have a normal instead of negative exponential distribution:

$$f(E) = \frac{1}{0.15\sqrt{2\pi}} \; e^{-1/2 \left(\frac{E - 0.6}{0.15}\right)^2} .$$

Most of the environmental events have a severity near the mean value, 0.6, and practically all severities lie between 0.15 and 1.05. Fig. 5 illustrates the exponential character of reliability as a function of operating time. It is noted that after a rapid early decrease in reliability the failure rate levels off. This is inherent in the conditions and constants of the model for in the early stages (small values of $\bar{n}$) the weaker mechanisms are failed. After this weeding out process has continued for a time, the remaining population is sufficiently failure resistant that the probability of occurrence of an environment severe enough to cause failure is very remote. The mean failure resistance of the remaining population then increases with elapsed operating time and the probability of failure per unit time decreases with increased use.

Model III. The conditions of this model are identical to those of Models Ia and II except that environmental severity is assumed to have a rectangular distribution. That is, the severity of individual environmental occurrences is equal-

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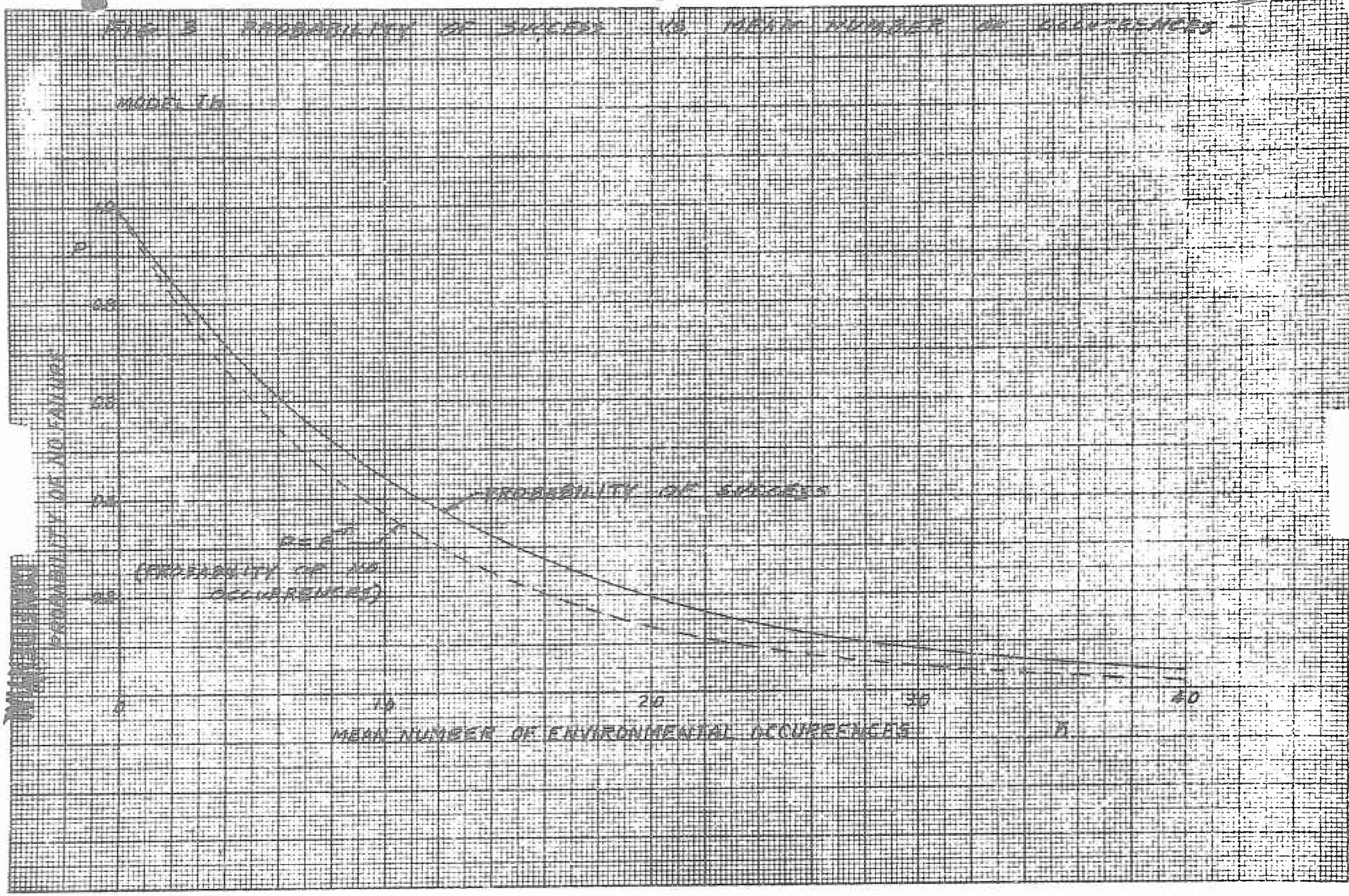

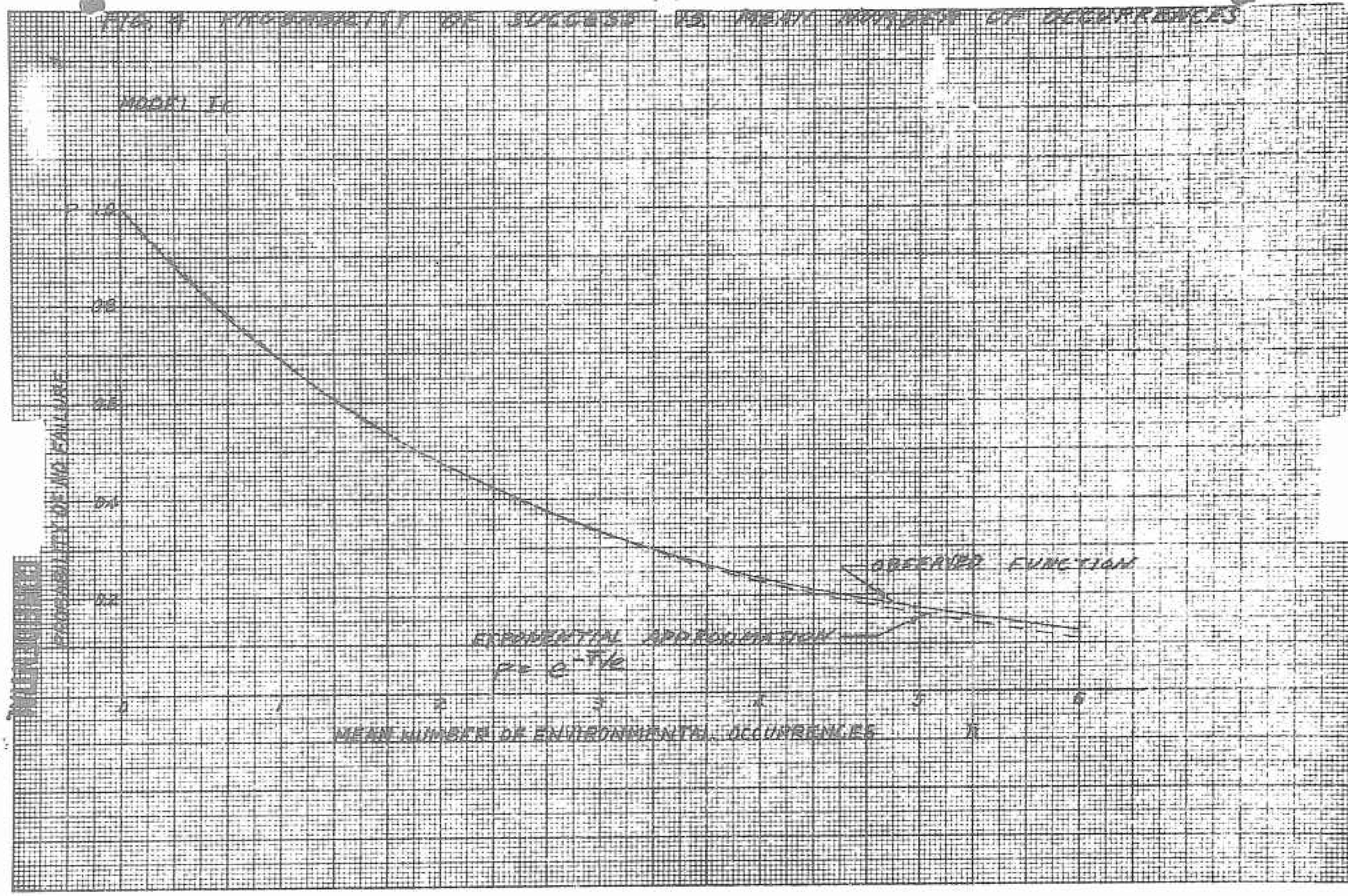
FIG. 4 PROBABILITY OF SUCCESS VS. MEAN NUMBER OF OCCURRENCES
MODEL Ic
P
1.0
0.8
0.6
0.4
0.2
PROBABILITY OF NO FAILURE
0
1
2
3
4
5
6
MEAN NUMBER OF ENVIRONMENTAL OCCURRENCES
$\bar{n}$
OBSERVED FUNCTION
EXPONENTIAL APPROXIMATION
$P = e^{-\bar{n}/k}$

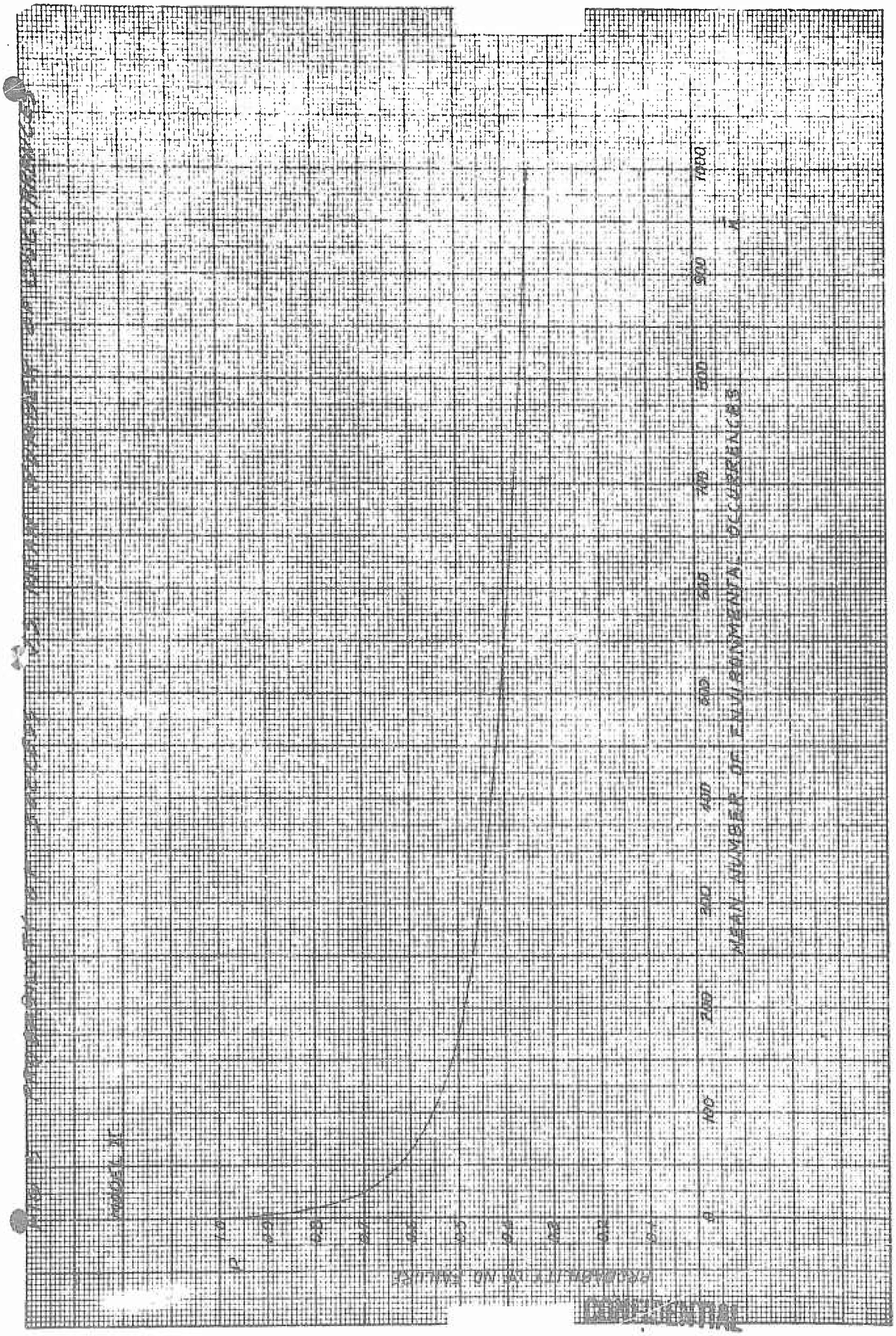
FIG 5 PROBABILITY OF SUCCESS VS MEAN NUMBER OF OCCURRENCES
P
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
PROBABILITY OF NO FAILURE
0
100
200
300
400
500
600
700
800
900
1000
MEAN NUMBER OF ENVIRONMENTAL OCCURRENCES
n

ly likely to have any value between an upper limit, 1.1 and lower limit, 0.1. Then

$$f(E) = \frac{1}{1.1 - 0.1}$$

and

$$\int_0^R f(E)dE = R - 0.1 \quad (0.1 \leq R \leq 1.1)$$

Fig. 6 represents the reliability of this model as a function of the length of the operating period. An exponential type of curve is observed which approaches a horizontal reliability asymptote of 0.31. Under the conditions of the model, it is impossible to fail that fraction of the mechanisms whose failure resistance is greater than the most severe environmental event (1.1).

Model IV. This model corresponds to Model Ia, having a normal distribution of failure resistance and a negative exponential distribution of environmental severity. However, the mean failure resistance is assumed to deteriorate with time instead of being constant. The deterioration rate is assumed to be proportional to the degree of deterioration at any time using the relation for mean failure resistance, $\bar{R} = \frac{1}{2}\ (3 - e^{0.001\ \bar{n}})$ and a constant standard deviation of failure resistance, 0.1, making

$$f(R) = \frac{1}{0.1\sqrt{2\pi}}\ e^{-1/2\left[\frac{R - \frac{1}{2}(3 - e^{0.001\ \bar{n}})}{0.1}\right]^2} .$$

The environmental severity distribution of this model is:

$$f(E) = \frac{1}{0.1}\ e^{-\frac{E}{0.1}} .$$

Fig. 7 indicates the rather normal tendencies of the probability of failure versus number of occurrences (time) of this model, which tendencies have been postulated as the distribution type attendant upon mechanisms that wear out. It can be shown that the deviation of the model distribution function from a normal is practically eliminated by the use of an arc tangent function to represent mean mechanism failure resistance:

$\bar{R} = a + b \tan^{-1} (c\ \bar{n} + d)$ in place of the exponential decay of mean failure resistance (a,b,c, and d are constants).

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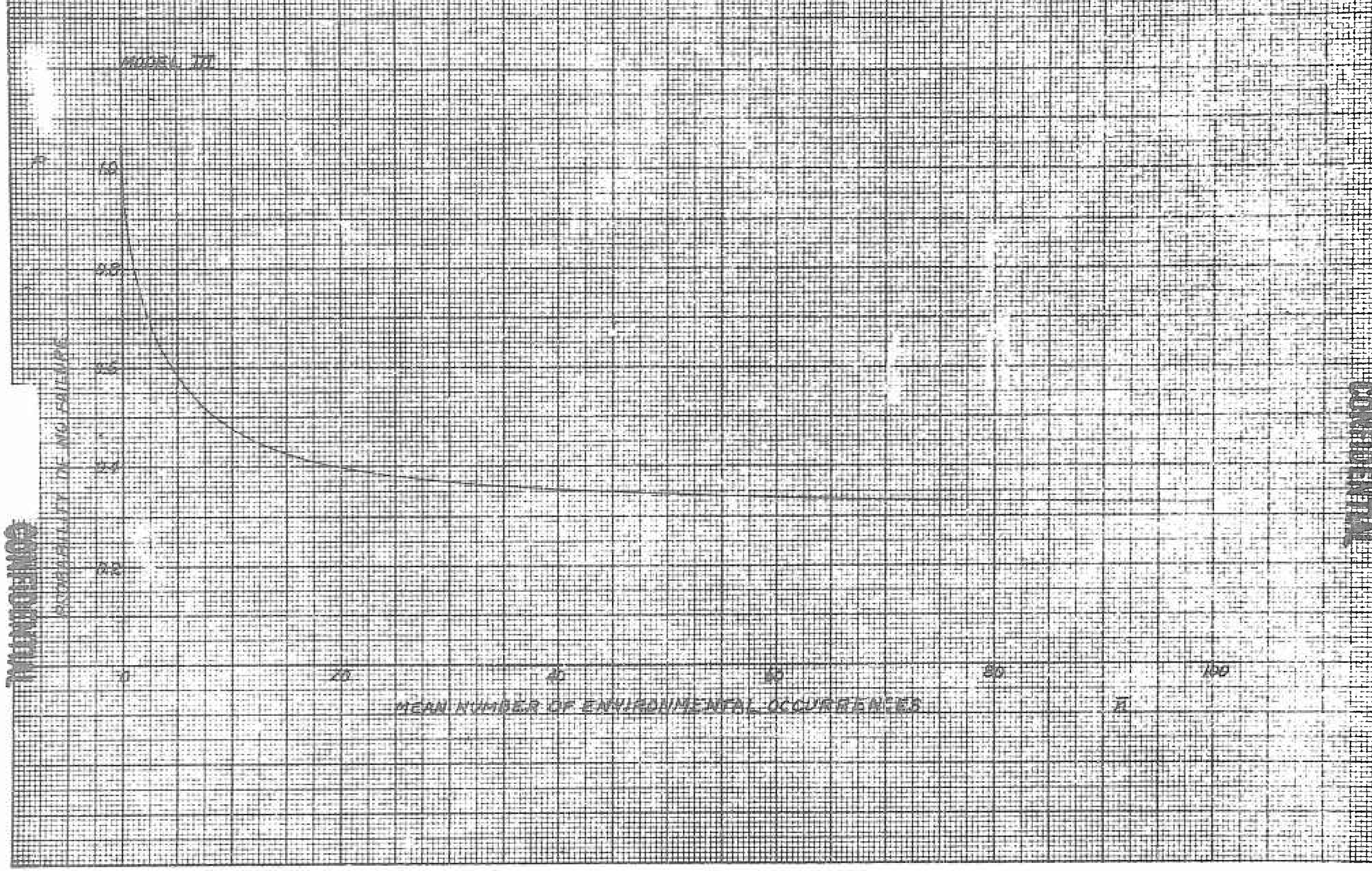
FIG. [illegible] PROBABILITY OF SUCCESS VS. MEAN NUMBER OF OCCURRENCES



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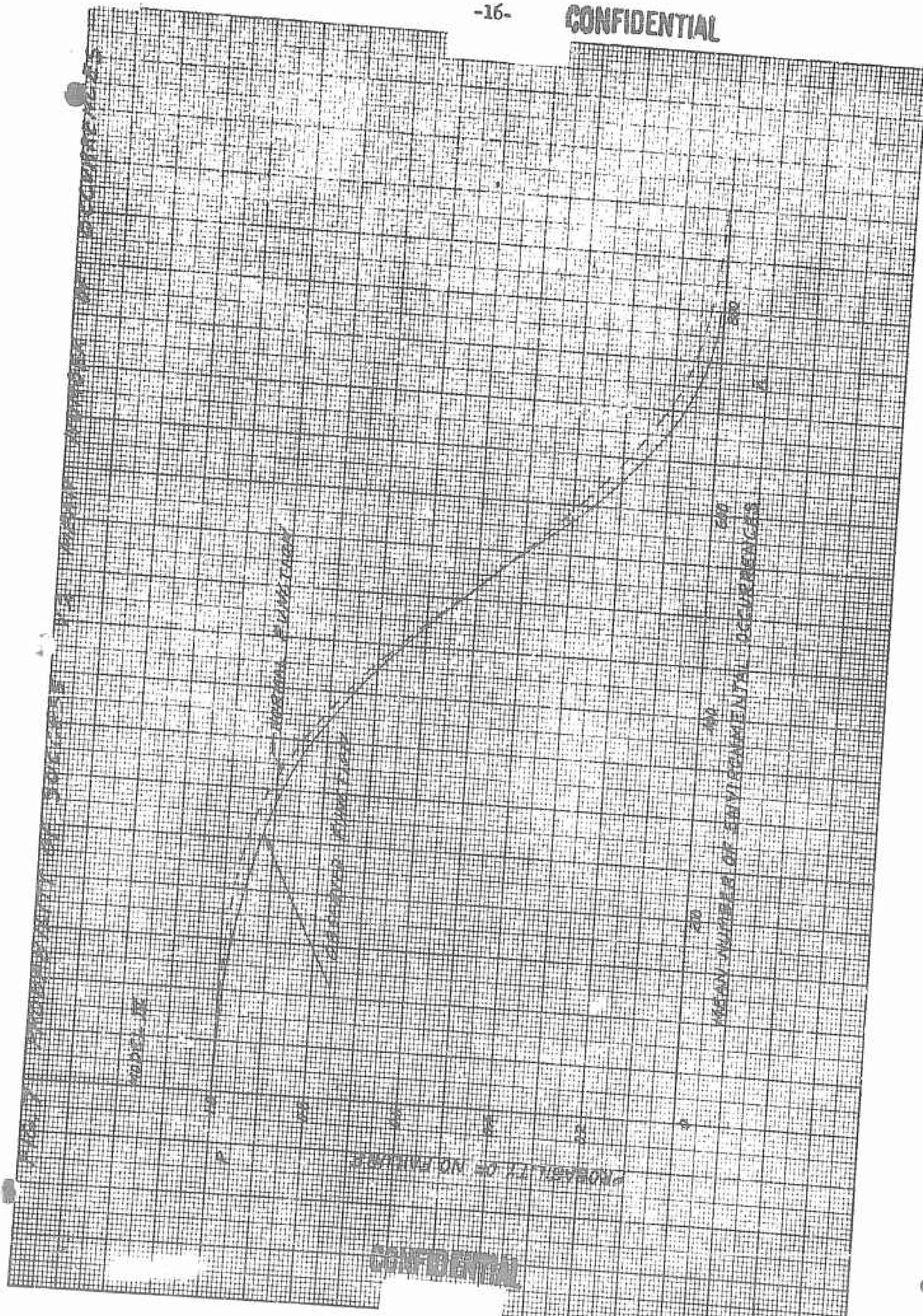
FIG. 7 PROBABILITY OF SUCCESS VS MEAN NUMBER OF OCCURRENCES
MODEL III
NORMAL FUNCTION
P
1.0
0.8
0.6
0.4
0.2
0
PROBABILITY OF NO FAILURE
200
400
600
800
MEAN NUMBER OF ENVIRONMENTAL OCCURRENCES

Model V. As a variant of Model IV the diminution of failure resistance is assumed to have an arbitrary but conceivable function of operating time:

$$\bar{\bar{R}} = 1 - \frac{1}{2} \frac{f(u)}{1 - \int_{-\infty}^{u} f(u)\, du}$$

where f(u) is the normal distribution function and u is evaluated as $u = 0.01\,\bar{n} - 3$.

This function is similar in derivation to the force of mortality used in actuarial calculations. The environmental severity used in this model is:

$$f(E) = \frac{1}{0.1} e^{-E/0.1}$$

The reliability of this model as a function of the number of occurrences shown in Fig. 8 has rather normal characteristics.

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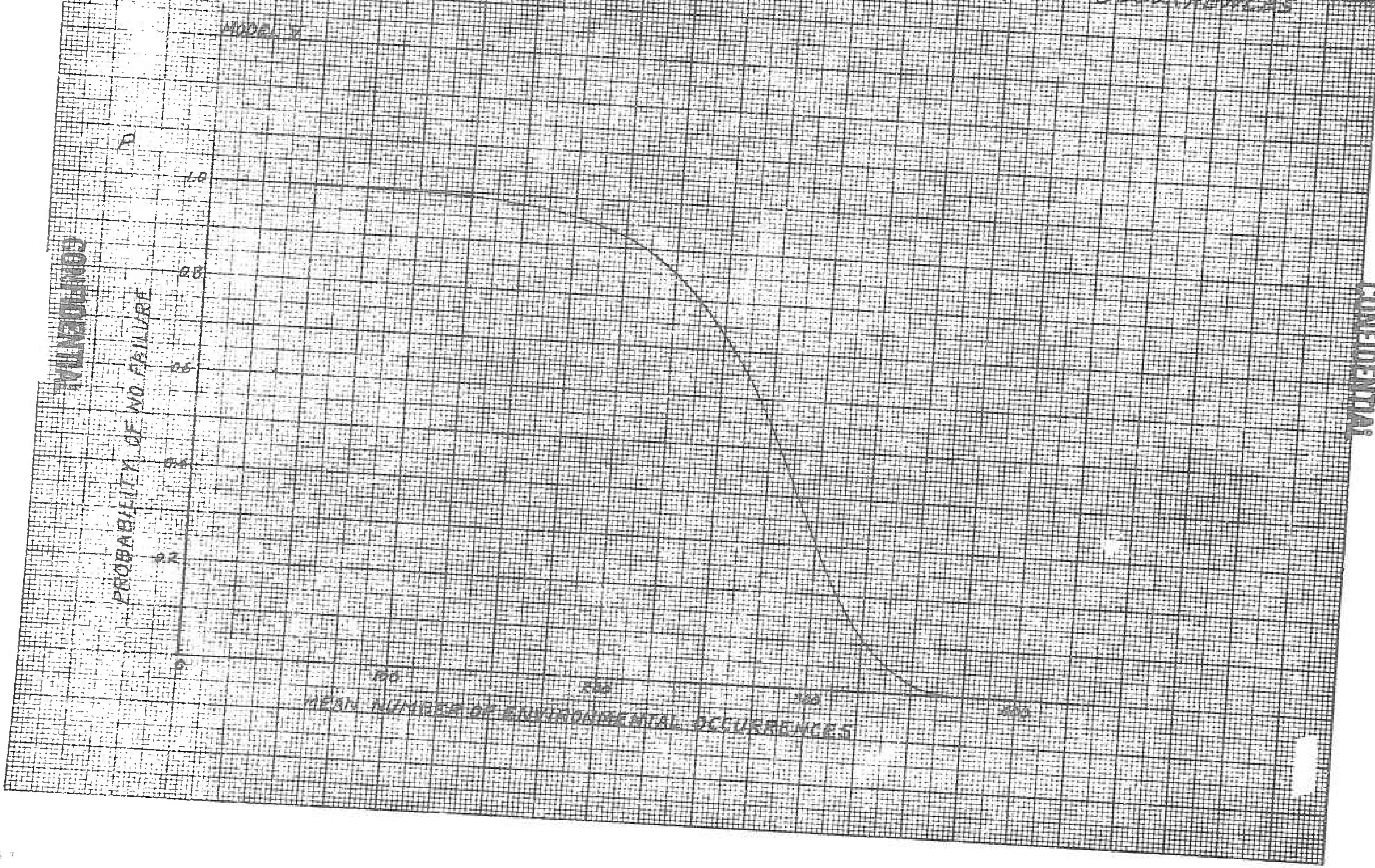

FIG. 8 PROBABILITY OF SUCCESS VS MEAN NUMBER OF OCCURRENCES

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TABLE OF MODELS USED

MODEL NO.	RESISTANCE	ENVIRONMENT	RESULTING FAILURE DISTRIBUTION
Ia	"Normal" distribution	Negative exponential. Least severe events occur most often. Mean environment low relative to mean resistance.	Envelope of chance failure curves of increasing resistance. Fig. 2, p. 9.
Ib	Same as Ia	Same as Ia except mean environment high relative to mean resistance.	Close to chance failure. Fig. 3, p. 11.
Ic	Same as Ia	Same as Ia except mean environment equal to mean resistance.	Close to chance failure. Fig. 4, p. 12.
II	Same as Ia	"Normal" distribution. Mean environment slightly lower than mean resistance.	Approximate chance failure modified by long lived survivors Fig. 5, p. 13.
III	Same as Ia	Rectangular distribution. Mean environ slightly lower than mean reisitance	Approximate chance failure modified by some immortal survivors. Fig. 6, p. 15.
IV	Same as Ia, except resistance deteriorates during operation, i.e., "wear out" occurs.	Same as Ia, except mean environment very low relative to mean starting resistance.	Resembles "Normal" distribution curve. Fig. 7, p. 16.
V	Same as Ia except resistance deteriorates in a manner similar to human life expectancy.	Same as Ia except mean environment very low relative to mean starting resistance.	Similar to Model IV. Fig. 8, p.18.

DISCUSSION

A. Variation of Failure Resistance With Time.

It is perhaps obvious that if all the factors affecting the probability of failure of survivors remained unchanged with the passage of time, the probability of survivor failure would also remain unchanged and a pure ''chance'' failure distribution would be the only possible outcome of the circumstances. The models examined display deviation from the ''chance'' hypothesis which must arise from a variation of the survivor failure resistance with time since the other factors are assumed invariable.

In the non-deteriorating models, as has been pointed out, the early failures tend to eliminate the weaker units. Thus the mean failure resistance of the survivors increases with the passage of time. The failure distribution of Model Ia is plotted in Fig. 9 together with the ''chance'' distributions corresponding to several points in the time history of Model Ia. The ''chance'' curves fit the model curve very closely in the region close to the tangent point. In fact the model failure distribution is the envelope of a series of ''chance'' curves whose mean times-to-failure increase with increasing time.

In contrast with increase in failure resistance of a population from natural selection with passage of time, the effect of time deterioration of failure resistance was noted in Models IV and V. In Fig. 10 it is apparent that the rather normal distribution curve of Model IV is the envelope of a series of "chance" curves whose slope-to-height ratios (failure susceptibilities) increase numerically with increasing time. It is interesting to speculate that the deterioration of the surviving population in an actual situation might compensate for the increase in mean failure resistance resulting from the elimination of the weaks. Should these opposite effects maintain the failure resistance distribution of the survivors identical to the original distribution, a pure ''chance'' failure distribution would result.

B. Simplifying Approximations.

Examination of the basic formula for probability of success (Eq. 7) making use of some simplifying assumptions sheds some light on the conditions that would produce an actual failure distribution similar to those arising from pure ''chance'' occurrences. It is recalled that a pure ''chance'' failure distribution evaluates the probability of success, P, during an operating period t as:

$$P = e^{-kt} \qquad (10)$$

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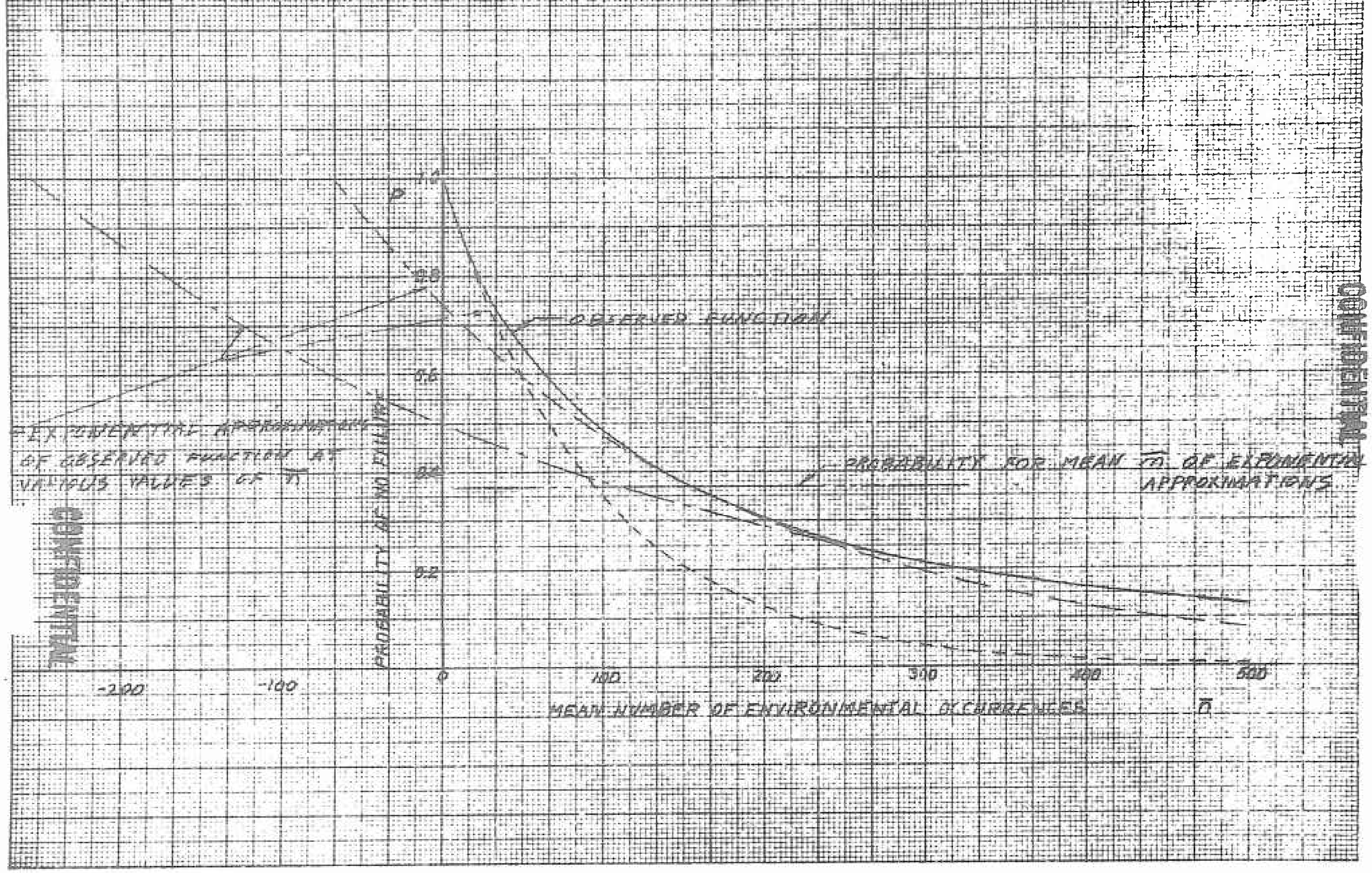
EXPONENTIAL APPROXIMATIONS OF PROBABILITY OF SUCCESS



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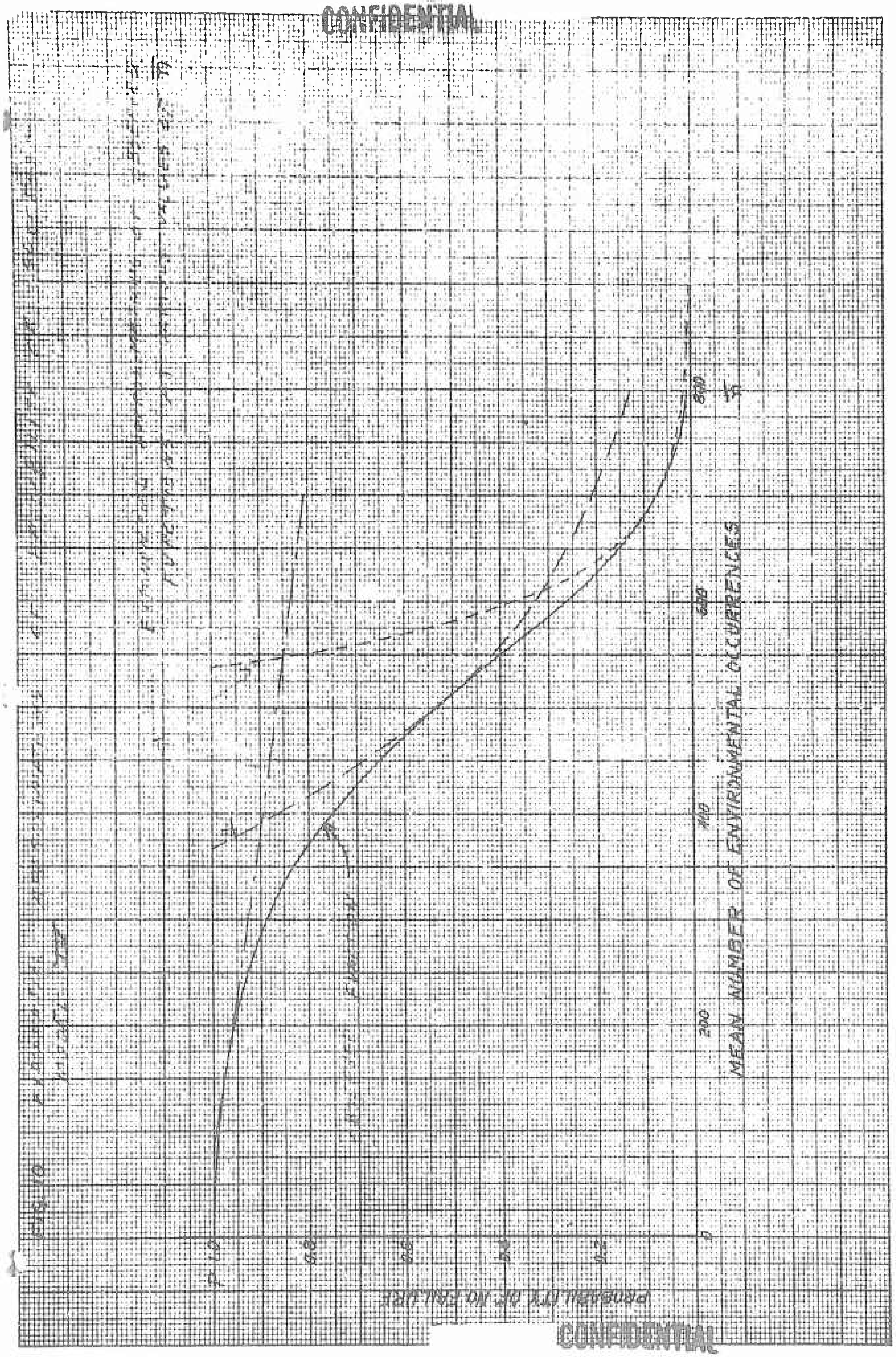

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Using f(E) to describe the distribution function of the environmental severity and f(R) to denote the distribution function of the mechanism failure resistance, equation 7 reduces to:

$$P = \int_{0}^{\infty} e^{-t/\bar{y}} \left[1 - \int_{0}^{R} f(E)\,dE\right] f(R)\,dR \text{ - - - - - - - - - - - - - - - - (11)}$$

The range of failure resistance employed in Model Ib is narrow compared to the range of environmental severity. Consequently, there is little more probability of failure for the weak mechanisms than for the strong ones and the distribution of survivors will not change much with time. Moreover the mean value of the failure resistance of survivors changes even less. In Model Ib the probability of failure for the average mechanism at the start is 0.819 per each environmental event. During an operating period t, the expected number of failure causing circumstances is: $\mathcal{E} = 0.819\ t/\bar{y}$ which produces a ''chance'' distribution of probability of success during time t for the average mechanism:

$$P = e^{-0.819\ t/\bar{y}} \text{ - (12)}$$

The probability of success of the average mechanism (Eq. 12) is practically indistinguishable from that of the distributed population shown in Fig. 3.

From equation 11, if all the mechanisms have the same failure resistance, R_1, the probability of success reduces to the ''chance'' form:

$$P = e^{-t/\bar{y}} \left[1 - \int_{0}^{R_1} f(E)\,dE\right] = e^{-kt} \text{ - - - - - - - - - - - - - - - - (13)}$$

regardless of what form the environmental distribution, f(E), may have. As pointed out above, a population of mechanisms displaying a narrow range of failure resistance may be reasonably approximated by the average mechanism of the population. Engineering design and quality control procedures tend to produce mechanisms with small ranges of failure resistance compared to environmental severity which should therefore produce ''chance'' failure distributions regardless of the environmental distribution, provided, of course, wear-out effects do not enter into the process.

Now consider the simplifying assumption that all environmental events are of equal severity E_1. Then the probability of a single environmental event being more severe than a particular failure resistance, R_1, is:

$$P\ (E > R) = 1 - \int_0^{R_1} f(E)\,dE = 1 \text{ if } 0 < R_1 < E_1$$

$$= 0 \text{ if } E_1 < R_1 < \infty$$

and from equation (11) the probability of success of the population of mechanisms during time t is:

$$P = e^{-t/\bar{y}} \int_{0 \doteq -\infty}^{E_1} f(R)\,dR \quad + \int_{E_1}^{\infty} f(R)\,dR \quad \text{- - - - - - - - - - - - - (14)}$$

$$= ce^{-t/\bar{y}} + 1-c$$

where c is the fraction of mechanisms having a failure resistance less than the environmental severity, E_1, and 1-c is the fraction having a failure resistance greater than E_1. The failure distribution under this simplifying assumption is a hybrid composed of a chance failure fraction, c, and an invulnerable fraction, 1-c. Models II and III produce distributions rather similar in character to that described by Eq. 14.

Should environmental severity have a distribution, the smallest value of which is greater than failure resistance of the strongest mechanism, any occurrence will fail any mechanism. The probability of success is then the probability of no environmental occurrences,

$$P = e^{-t/\bar{y}}$$

which also is a pure ''chance'' failure distribution.

Model Ic illustrates the case in which the distributions of failure resistance and environmental severity overlap each other to a very great extent. While the actual distribution deviates appreciably from the approximation based on the crude assumption that all the mechanisms exhibit the mean value of failure resistance, the percent of error arising from the approximation is surprisingly small.

C. Improving Reliability by Artificial Selection

Some general inferences can be drawn regarding an artificial selection process to improve reliability in the light of the models examined. Such an artificial selection process is designed to eliminate the weaker individuals from the population by subjecting the whole population to a pre-use environment which will fail the poorer specimens. If the failure resistance of the survivors is greater than that of the unselected population, an improvement in reliability results. The failure distribution of the select group will exhibit the same general characteristics as that of the original population but the mean time to failure will tend to be longer and the probability of failure during equal operating periods will be smaller.

The criterion of an effective selection process is that it either fail or indicate the weak units and further that it does not seriously weaken the survivors of the process (little deterioration during selection). It follows that the skimming off of a random sample is an ineffictive process. Service conditions tend to tend to large variations in environmental severity like the conditions of the models examined. Selection under service conditions will usually produce little improvement in the selected population as the selection process will not fail some of the weak units and will fail some of the strong ones, thus, skimming off a random sample. Effective selection results from a closely controlled environment designed to fail all the mechanisms weaker than the desired value and pass the stronger ones. A further requirement of the selection environment is that it does not weaken the failure resistance of the survivors. A standard "running in" test on aircraft engines is an example of a controlled elimination of weak units.

CONCLUSIONS

1. It appears that a negative exponential or "chance" type of failure distribution approximately describes reasonable combinations of environment and mechanism having the distribution types assumed, provided the mechanism does not deteriorate in failure resistance. During the short operating periods of interest in guided missile reliability prediction, the assumption of no deterioration appears quite valid. Also for short operating periods an exponential or "chance" approximation is very accurate.

2. The introduction of failure resistance deterioration into the situation changes the characteristics of the failure distribution with time to a rather normal appearing function, at least for reasonable deterioration assumptions.

3. Artificial selection processes may be set up to improve reliability by eliminating weak numbers and thus increase the average failure resistance of the population. A closely controlled environment that does not seriously impair the failure resistance of the survivors appears best suited to the process.

REFERENCES

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LIST OF SYMBOLS

a	=	a constant
b	=	a constant
c	=	a constant
d	=	a constant or a differential element of a variable as dx
e	=	the constant 2.7183
E	=	environmental severity
$\bar{E}$	=	mean environmental severity
f( )	=	a function of the parenthesized variable usually the distribution function
f(u)	=	normal distribution function of u = $\frac{1}{\sqrt{2\pi}} e^{-1/2\, u^2}$
k	=	a constant
n	=	number of environmental events occurring during an operating period
$\bar{n}$	=	average number of environmental events occurring during an operating period
P	=	probability
Q	=	ordinate value used in probability computation
R	=	failure resistance
$\bar{R}$	=	mean failure resistance
S	=	abscissa value
t	=	operating period
$\bar{y}$	=	average time interval between environmental events
1(subscript)	=	a particular value of the variable as R_1
$\mathcal{E}$	=	expected number of events or failures
σ	=	standard deviation